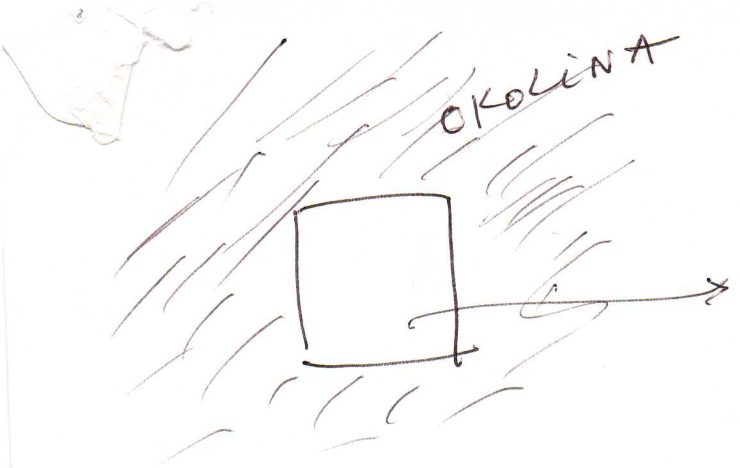
FENOMENOLOŠKA TERMODINAMIKA



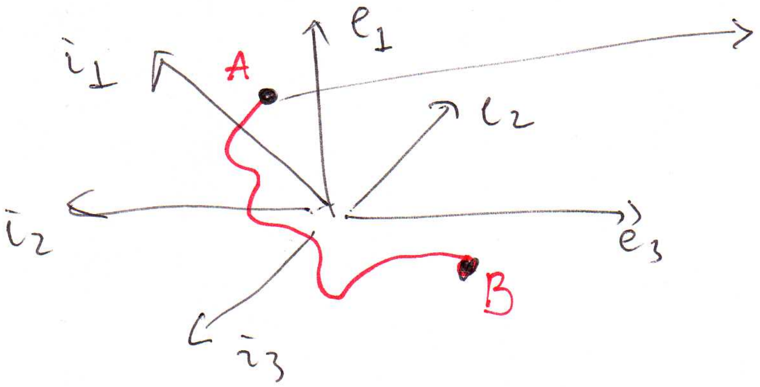
Termodinamički sistem
(veliki broj unutrašnjih stepeni slobode)

TD parametri : odnos sistema sa spoljašnjom okolinom

- spoljašnji (e_K)
- unutrašnji (i_K)

- ekstenzivni (V, C_P, C_V)
- intenzivni (T, P, ρ)

PROSTOR TD parametara – prostor stanja



TD stanje
(skup svih parametara neophodnih da makroskopski [illegible] osobine sistema i odnos sistema sa okolinom)

TD ravnoteža je TD stanje koje se ne menja sa vremenom

J-na stanja je funkcionalna veza među elementima onog skupa parametara koji je [illegible] i dovoljan za opis stanja sistema.

Npr. za termomehanički sistem j-na stanja glasi

$$f(p,V,T)=0$$

F-je stanja: U, F, G, H, S ....

$$\oint_C dU=0, \quad \oint_C dF=0 \;....$$

$\downarrow$ putanja u prostoru TD parametara

F-je stanja imaju egzaktne diferencijale

$$dA \Longleftrightarrow \oint_C dA=0$$

Ne-egzaktni diferencijal δA

Rad TD sistema

$$\delta W=\sum_{k=1}^{N} A_k\, de_k \quad ::: \quad \vec{F}(\vec{r})\, d\vec{r}, \quad pdV$$

A_k $\downarrow$ generalisane sile

$U=U(T, e_1, \ldots e_m)$ – kalorična j-na stanja $\Longleftrightarrow U=U($

$U=U($

$U=U($

$A_k=A_k(T, e_1, \ldots e_m)$ – termička j-na stanja

$\hookrightarrow p=p(T,V)$

Proizvoljna f-ja

$f(x,y)$

diferencijal $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

uslov za tot. dif.

$$\boxed{\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}} \iff \boxed{\oint df = 0}$$

0 ZAKON TD: STANJE RAVNOTEŽE ZA SISTEME koji su dovedeni u međusobni kontakt postoji i važi osobina tranzitivnosti: Ako je

$$A \overset{TD}{=} B$$
$$B \overset{TD}{=} C$$
$$\Downarrow$$
$$A \overset{TD}{=} C$$

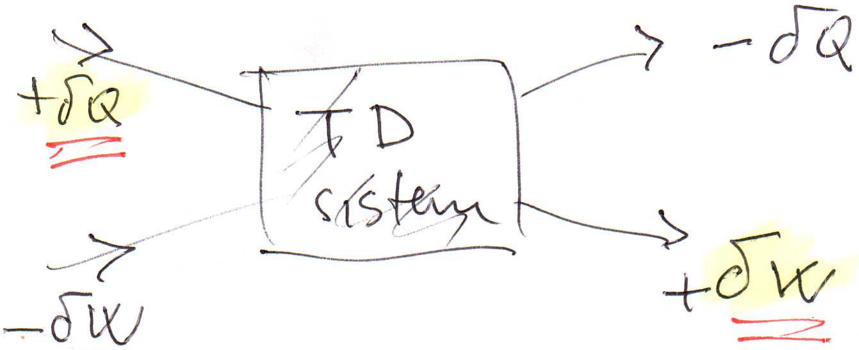
↓

Egzistencija stanja TD ravnoteže

→ **Koncept temperature**

I zakon TD (zakon o održanju energije)

$$du = \delta Q - \delta W$$



$$du = (+\delta Q) - (+\delta W)$$
$$du = (+\delta Q) - (-\delta W)$$
$$du = (-\delta Q) - (+\delta W)$$
$$du = (-\delta Q) - (-\delta W)$$

II zakon TD govori o smeru TD procesa i uvodi ENTROPIJU

$$dS = \frac{\delta Q}{T}$$

U stanju TD ravnoteže $dS \geq 0$

I + II zakon TD za termomehaničke sisteme

$$TdS = dU + pdV$$

III zakon TD: Apsolutnu nulu je nemoguće dostići

TD potencijali: $\Leftrightarrow$ f-ja stanja $\rightarrow$ kalorična j-na stanja

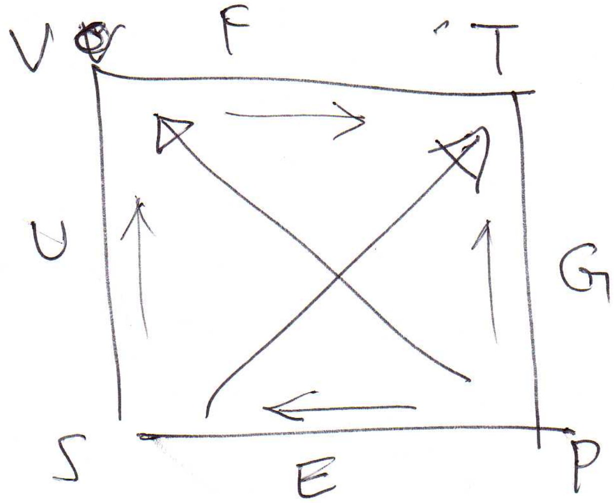
$U = U(S,V)$ ali npr. $U(T,V)$ nije TD potencijal već f-ja stanja so

$$G(P,T) = U - TS + pV$$

$$F(V,T) = U - ST$$

$$E(S,P) = U + pV$$

Bornov TD četvorougao



$$dF = -pdV - SdT$$

$$dG = -SdT + Vdp$$

$$dE = TdS + Vdp$$

$$dU = TdS - pdV$$

Maksvelove TD j-ne

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$$

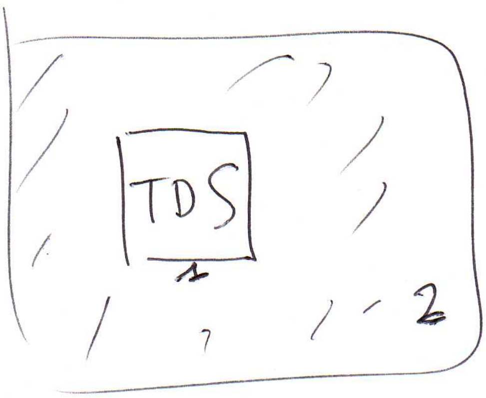
$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

Ako sistem može da razmenjuje čestice onda se dif. formama dodaje član

$$\boxed{\mu \, dN}$$

hemijski potencijal → rad koji je potrebno uložit da se sistemu doda ili oduzme čestica (konstit



uslovi ravnoteže

$$\boxed{\begin{aligned} T_1 &= T_2 \\ P_1 &= P_2 \\ \mu_1 &= \mu_2 \end{aligned}}$$

Pojam Jakobijana $\Leftarrow$ Višestruki integrali

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \left(\frac{\partial u}{\partial x}\right)_y & \left(\frac{\partial v}{\partial x}\right)_y \\ \left(\frac{\partial u}{\partial y}\right)_x & \left(\frac{\partial v}{\partial y}\right)_x \end{vmatrix} \quad ; \quad du\,dv = |J|\,dx\,dy$$

$v = y$

$$\frac{\partial(u,v)}{\partial(x,y)} \equiv \left(\frac{\partial u}{\partial x}\right)_y$$

OSOBINE: (dokazuju se po definiciji)

1. $$\frac{\partial(u,v)}{\partial(x,y)} = -\frac{\partial(v,u)}{\partial(x,y)}$$

2. $$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)}\,\frac{\partial(r,s)}{\partial(x,y)}$$

3. $$\frac{\partial(u,v)}{\partial(x,y)} = \left[\frac{\partial(x,y)}{\partial(u,v)}\right]^{-1}$$

Obnoviti: diferencijalni račun f-ja više promenljivih i linijski integral

F-JE ODZIVA

$$\left.\begin{aligned} C_v &= \left(\frac{\partial u}{\partial T}\right)_v \\ C_p &= \left(\frac{\partial u}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p \end{aligned}\right\} \quad C_x = \left(\frac{\delta Q}{dT}\right)_x$$

$$\left.\begin{matrix} C_p \\ C_v \\ K_T \\ K_S \end{matrix}\right\} > 0$$

$$\alpha_p \lessgtr 0$$

$K_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ izotermska kompresibilnost

$\alpha_p = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$ koeficijent toplotnog širenja

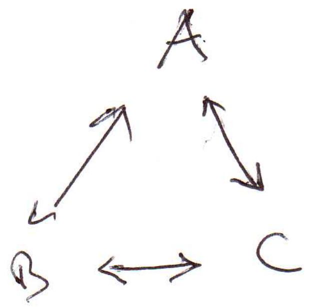
$K_S = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$ adijabatska kompresibilnost

Domaći:

Neka je data diferencijalna forma

$$dQ = A(x,y,z)\,dx + B(x,y,z)\,dy + C(x,y,z)\,dz$$

Kako glase uslovi koji treba da budu ispunjeni da bi dQ bilo totalni diferencijal?



$$\left(\frac{\partial A}{\partial y}\right) = \left(\frac{\partial B}{\partial x}\right)$$

$$\left(\frac{\partial A}{\partial z}\right) = \left(\frac{\partial C}{\partial x}\right)$$

$$\left(\frac{\partial B}{\partial z}\right) = \left(\frac{\partial C}{\partial y}\right)$$

pri čemu je je $A = \frac{\partial Q}{\partial x}$, $B = \frac{\partial Q}{\partial y}$, $C = \frac{\partial Q}{\partial z}$

1 Neka su date f-je $z = z(x,y)$ i $w = w(x,y)$

Pokazati da za njih važe sledeće ~~obrasci~~ relacije

a) $\left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{\left(\frac{\partial z}{\partial y}\right)_x}$ (relacija recipročnosti)

b) $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = -1$ Eulerova ciklična relacija

c) $\left(\frac{\partial z}{\partial w}\right)_x = \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial w}\right)_x$

d) $\left(\frac{\partial z}{\partial x}\right)_w = \left(\frac{\partial z}{\partial x}\right)_y + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_w$

e) $\left(\frac{\partial z}{\partial x}\right)_w \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = 1.$

$\left.\begin{array}{l} \text{I } \underline{12 \text{ poena}} \\ \text{II } \underline{12 \text{ poena}} \end{array}\right\} 2X$

$z = z(x,y)$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

Iz $z = z(x,y)$ se u principu može dobiti

$y = y(x,z)$, pa važi

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz$$

Zameniti ~~ovo~~ dz iz gornjeg izraza u donji pa sledi

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$0 = \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y\right] dx + \left[\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial y}\right)_x - 1\right] dy$$

~~x, y nezavisne promenljive~~

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial y}\right)_x = 1 \Rightarrow \boxed{\left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{\left(\frac{\partial z}{\partial y}\right)_x}}$$

(a)

Važi ciklično $x \to y \to z$ (proveriti) u z dx

$$\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = 0$$

$$\left(\frac{\partial y}{\partial x}\right)_z = - \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y$$

$$\frac{1}{\left(\frac{\partial x}{\partial y}\right)_z} = - \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y$$

$$\boxed{-1 = \left(\frac{\partial y}{\partial x}\right)_z \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x}$$ (b)

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$y = y(w, x) \Leftarrow \quad w = w(x, y)$$

$$dy = \left(\frac{\partial y}{\partial w}\right)_x dw + \left(\frac{\partial y}{\partial x}\right)_w dx$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x \left[\left(\frac{\partial y}{\partial w}\right)_x dw + \left(\frac{\partial y}{\partial x}\right)_w dx\right]$$

$$dz = \left[\left(\frac{\partial z}{\partial x}\right)_y + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_w\right] dx + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial w}\right)_x dw$$

Budući da važi

$$z = z(x,y) \qquad i \quad y = y(w,x)$$

onda je i

$$z = z(x, y(w,x)) = z(x,w)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw$$

Implicitno se koristi pojam invarijantnosti forme diferencijala!

Poređenjem se dobija

$$\left(\frac{\partial z}{\partial x}\right)_w = \left(\frac{\partial z}{\partial x}\right)_y + \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_w \qquad \text{(d)}$$

$$\left(\frac{\partial z}{\partial w}\right)_x = \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial w}\right)_x \qquad \text{(c)}$$

DOMAĆI:

Iz c), koristeći a) imamo

$$\left(\frac{\partial z}{\partial w}\right)_x \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial w}{\partial y}\right)_x = 1$$

tj.

$$\left(\frac{\partial z}{\partial w}\right)_x \left(\frac{\partial w}{\partial y}\right)_x \left(\frac{\partial y}{\partial z}\right)_x = 1$$

i cikličnom zamenom dobijamo izraz pod **(e)**

$$\begin{tikzcd} & x \arrow[dr] & \\ w \arrow[ur] & & y \arrow[dl] \\ & z \arrow[ul] & \end{tikzcd}$$

$$w \to x \to y \to z$$

2. Kako glasi veza izmedju kaloričke j-ne stanja $u=u(T,V)$ i termičke j-ne sta[nja] $p=p(T,V)$ za termomehanički sistem

$$du = T\,ds - p\,dV$$

a) Preko unutrašnje ene[rgije]
b) Preko slobodne energ[ije]

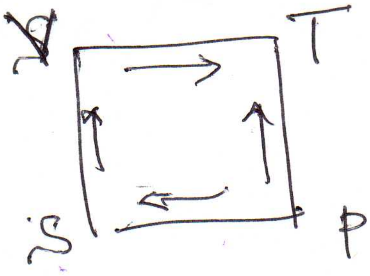
$$u = u(V,T)$$

$$s = s(V,T)$$

$$\left(\frac{\partial u}{\partial V}\right)_T dV + \left(\frac{\partial u}{\partial T}\right)_V dT = T\left(\left(\frac{\partial s}{\partial V}\right)_T dV + \left(\frac{\partial s}{\partial T}\right)_V dT\right) - p\,dV$$

$$\left[\left(\frac{\partial u}{\partial V}\right)_T \cancel{+} - T\left(\frac{\partial s}{\partial V}\right)_T + p\right] dV + \left[\left(\frac{\partial u}{\partial T}\right)_V - T\left(\frac{\partial s}{\partial T}\right)_V\right] dT = 0$$

$$\left(\frac{\partial u}{\partial V}\right)_T - T\left(\frac{\partial s}{\partial V}\right)_T + p = 0$$



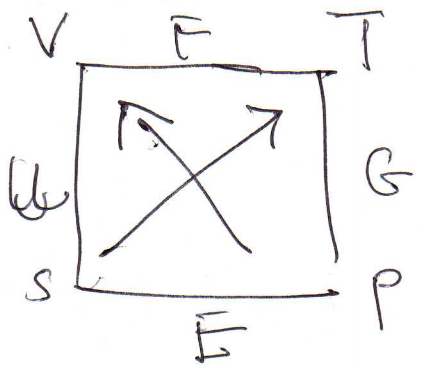
$$\Longrightarrow \left(\frac{\partial s}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial u}{\partial V}\right)_T = -p + T\left(\frac{\partial p}{\partial T}\right)_V$$

Veza je neholonomnog karaktera

Domaći:
Šta određuje član uz dT u gornjem izrazu?

Takodje i



$$\Rightarrow \quad dF = -p\,dV - S\,dT$$

$$\Downarrow$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$F = U - ST$$

$$\left(\frac{\partial F}{\partial V}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T - T\left(\frac{\partial S}{\partial V}\right)_T$$

$$-p = \left(\frac{\partial U}{\partial V}\right)_T - T\left(\frac{\partial p}{\partial T}\right)_V$$

$$\Downarrow$$

$$\boxed{\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p}$$

V. pozadi!

Šta je sa drugom neholonomnom vezom između kal. i ter. jne stanja

O FUNKCIJAMA ODZIVA

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$$

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$ koeficijent toplotnog širenja

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$ izotermska kompresibilnost

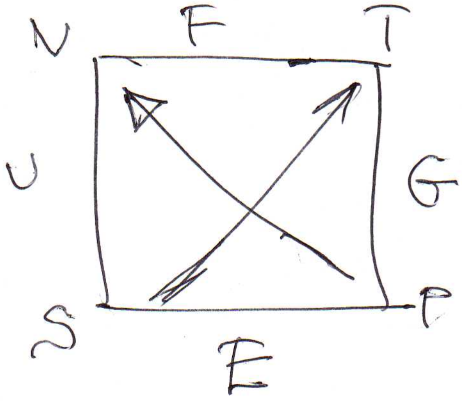
$$\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$ adijabatska kompresibilnost

$$\boxed{\xi_X = \frac{1}{V}\left(\frac{\Delta V}{\Delta Y}\right)_X}$$

opšta forma

$\frac{\Delta V}{\ldots}$

3. Iz uslova za totalni diferencijal diferencija-lnih formi za U, F, G i E izvesti odgovara-juće Meksvelove TD j-ne



$$du = -p\,dv + T\,ds$$

$$p = -\left(\frac{\partial u}{\partial v}\right)_s \qquad T = \left(\frac{\partial u}{\partial s}\right)_V$$

$$\frac{\partial^2 u}{\partial v \partial s} = \frac{\partial^2 u}{\partial s \partial V}$$

$$\left(\frac{\partial}{\partial v}\left(\frac{\partial u}{\partial s}\right)_V\right)_s = \left(\frac{\partial}{\partial s}\left(\frac{\partial u}{\partial v}\right)_s\right)_V$$

$$\boxed{\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_V}$$

$$dF = -p\,dv - S\,dT$$

$$p = -\left(\frac{\partial F}{\partial v}\right) \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$\frac{\partial^2 F}{\partial v \partial T} = \frac{\partial^2 F}{\partial T \partial v}$$

$$\left(\frac{\partial}{\partial v}\left(\frac{\partial F}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial v}\right)_T\right)_V$$

$$-\left(\frac{\partial S}{\partial v}\right)_T = -\left(\frac{\partial p}{\partial T}\right)_V$$

$$\boxed{\left(\frac{\partial S}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V}$$

4. Uspostaviti vezu između f-ja odziva α_p, K_T i β_V definisanih relacijama:

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p , \quad K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \quad i$$

$$\beta_V = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_V$$ koristeći osobinu Jakobijana da je

$$\frac{\partial(u,x)}{\partial(v,x)} = \left(\frac{\partial u}{\partial v}\right)_x ,$$

Primedba: piše se i

$$\frac{D(u,x)}{D(v,x)} = \left(\frac{\partial u}{\partial v}\right)_x$$

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V}\frac{\partial(V,p)}{\partial(T,p)} = -\frac{1}{V}\frac{\partial(V,p)}{\partial(p,T)}$$

$$= -\frac{1}{V}\frac{\partial(V,p)}{\partial(V,T)}\frac{\partial(V,T)}{\partial(p,T)} = -\frac{1}{V}\frac{\partial(p,V)}{\partial(T,V)}\frac{\partial(V,T)}{\partial(p,T)}$$

$$= -\frac{1}{V}\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{\cancel{V}}(p\beta_V)(-\cancel{V}K_T)$$

$$= p\beta_V K_T$$

$$\boxed{\frac{\alpha_p}{K_T} = p\beta_V}$$

Varijacija na temu za kolokvijum, pismeni?

5. Koristeći formalizam Jakobijana, izraziti κ_T/κ_S u f-ji toplotnih kapaciteta C_p i C_V

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T, \quad \kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

$$\frac{\kappa_T}{\kappa_S} = \frac{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T}{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S} = \frac{\dfrac{\partial(V,T)}{\partial(p,T)}}{\dfrac{\partial(V,S)}{\partial(p,S)}}$$

$$= \frac{\partial(V,T)}{\partial(V,S)}\,\frac{\partial(p,S)}{\partial(p,T)} = \left(\frac{\partial T}{\partial S}\right)_V \left(\frac{\partial S}{\partial T}\right)_p$$

$$= \frac{\left(\frac{\partial S}{\partial T}\right)_p}{\left(\frac{\partial S}{\partial T}\right)_V} = \frac{T\left(\frac{\partial S}{\partial T}\right)_p}{T\left(\frac{\partial S}{\partial T}\right)_V} = \frac{C_p}{C_V}$$

Za domaći: Izraziti $\frac{C_p}{C_V}$ u f-ji κ_T i κ_S!

Za domaći: Pokazati da je unutrašnja energija sistema čija termička j-na ima oblik $p = f(V)\,T$, nezavisna od zapremine. Navesti primer za to. (Koristiti vezu između $U(V,T)$ i $p(V,T)$)

6 Utvrditi da li sledeće diferencijalne forme predstavljaju egzaktne diferencijale!

a) $dA = \left(\frac{2}{V} + \frac{V}{T}\right) dT + \left(\frac{3V}{T} + 2\right) dV$

b) $dB = p\,dV - V\,dp$

c) $dC = p^2 dT - \frac{T^2}{p} dp$

ovakav tip zadatka za kolokvijum!

d) $dF = 2TV\,dT + T^2 dV$

e) $dG = 3T(Tp-2)\,dT + (T^3+2p)\,dp$

---

a) $\left(\frac{\partial A}{\partial T}\right)_V = \frac{2}{V} + \frac{V}{T}$

$\left(\frac{\partial A}{\partial V}\right)_T = \frac{3V}{T} + 2$

Uslov $\quad \frac{\partial^2 A}{\partial V \partial T} = \frac{\partial^2 A}{\partial T \partial V}$

$$\left(\frac{\partial}{\partial V}\left(\left(\frac{\partial A}{\partial T}\right)_V\right)\right)_T = \left(\frac{\partial}{\partial T}\left(\left(\frac{\partial A}{\partial V}\right)_T\right)\right)_V$$

$$\left(\frac{\partial}{\partial V}\left(\frac{2}{V} + \frac{V}{T}\right)\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{3V}{T} + 2\right)\right)_V$$

$$-\frac{2}{V^2}+\frac{1}{T} \neq -\frac{3V}{T^2}$$

dA nije totalni diferencijal

$$\oint_C dA \neq 0$$ C – putanja u prostoru p–V–T

b) $dB = p\,dV - V\,dp \Rightarrow \left(\frac{\partial B}{\partial V}\right)_p = p$

$$\frac{\partial^2 B}{\partial p \partial V} = \frac{\partial^2 B}{\partial V \partial p} \qquad \left(\frac{\partial B}{\partial p}\right)_V = -V$$

$$\left(\frac{\partial}{\partial p}\left(\left(\frac{\partial B}{\partial V}\right)_p\right)\right)_V = \left(\frac{\partial}{\partial V}\left(\left(\frac{\partial B}{\partial p}\right)_V\right)\right)_p$$

$$\left(\frac{\partial p}{\partial p}\right)_V \neq -\left(\frac{\partial V}{\partial V}\right)_p$$

dB nije totalni diferencijal

$$\oint_C dB \neq 0$$ C – putanja u prostoru P–V–T

c)

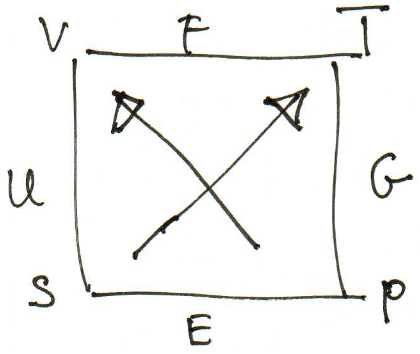
d) Domaći

e)

21. Jednačina stanja gasa je data sledećom jednačinom

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT.$$

Gas trpi izotermnu promenu zapremine sa V_1 na V_2. Izračunati promenu slobodne energije gasa, F. Kako glasi ΔF za idealni gas?



$$dF=-pdV-SdT$$

$$T=\text{const}$$

$$dF=-pdV$$

$$\Delta F=\int_A^B dF=\int_{V_1}^{V_2}-pdV$$

$$p=\frac{RT}{V-b}-\frac{a}{V^2}\Rightarrow -p=\frac{a}{V^2}-\frac{RT}{V-b}$$

$$\Delta F=\int_{V_1}^{V_2}\left(\frac{a}{V^2}-\frac{RT}{V-b}\right)dV$$

$$=a\int_{V_1}^{V_2}\frac{dV}{V^2}-RT\int_{V_1}^{V_2}\frac{dV}{V-b}$$

$$=-a\frac{1}{V}\Big|_{V_1}^{V_2}-RT\ln(V-b)\Big|_{V_1}^{V_2}$$

$$=-a\left(\frac{1}{V_2}-\frac{1}{V_1}\right)-RT\ln\frac{V_2-b}{V_1-b}$$

$$= -a\frac{V_1 - V_2}{V_1 V_2} - RT\ln\frac{V_2 - b}{V_1 - b}$$

$$\Delta F = a\frac{V_2 - V_1}{V_1 V_2} + RT\ln\frac{V_1 - b}{V_2 - b}$$

Za idealni gas $a \to 0$, $b \to 0$

$$\Delta F = RT\ln\frac{V_1}{V_2}$$

Primedba:
Varijacija na temu za kolokvijum!
Neka druga je na stanju, npr!
Или неки други да потвикујала?

8. Termička i kalorička j-na stanja su za neki TD sistem date izrazima

$$p = \frac{AT^3}{V}$$

i

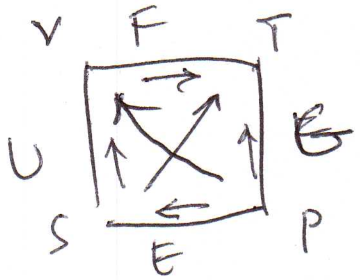
$$U = BT^n \ln \frac{V}{V_0} + f(T)$$

respektivno. A, B, n i V_0 su konstante i $f(T)$ zavisi samo od temperature. Odrediti B i n.

Koristiti vezu

$$\left(\frac{\partial U}{\partial V}\right)_T = -p + T\left(\frac{\partial p}{\partial T}\right)_V \qquad (*)$$

Izvođenje veze preko slobodne energije



$$F = U - ST \qquad (**)$$

$$dF = -p\,dV - S\,dT$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T \; ; \; \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$(**) \Big/ \frac{\partial}{\partial V} \Rightarrow \left(\frac{\partial F}{\partial V}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T - T\left(\frac{\partial S}{\partial V}\right)_T$$

$$-p = \left(\frac{\partial U}{\partial V}\right)_T - T\left(\frac{\partial p}{\partial T}\right)_V \Rightarrow (*)$$

Dakle, koristeći (*) i izvode

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{3A}{V} T^2$$

$$\left(\frac{\partial u}{\partial V}\right)_T = B T^n \frac{\cancel{V_0}}{V} \frac{1}{\cancel{V_0}} = \frac{BT^n}{V}$$

biće

$$T \frac{3A}{V} T^2 - \frac{AT^3}{V} = \frac{BT^n}{V}$$

$$\frac{2AT^3}{V} = \frac{BT^n}{V} \Rightarrow \begin{array}{l} B = 2A \quad i \\ n = 3 \end{array}$$

Primedba: Za kolokvijum, varijacija na temu. Na primer,

$$p = \frac{AT^4}{V} \qquad u = BT^4 \ln \frac{V}{V_0} + f(T)$$

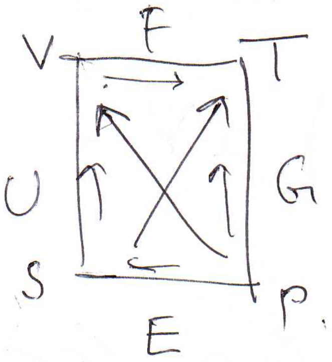
ili

$$p = \frac{AT^4}{4} + f(V) \qquad u = BT^3 \ln \frac{V}{V_0}, \quad \text{itd.}$$

Za neko čvrsto telo eksperimentalno je utvrđeno da u oblasti pritisaka $p_1 \le p \le p_2$ njegov koeficijent izobarnog termičkog širenja ispoljava sledeći tip zavisnosti

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V}\left(a+bp+cp^2\right)$$

gde su a, b i c konstante. Za koliko će se promeniti entropija ovog tela pri izotermnom sabijanju od p_1 do p_2?



Uočiti: $\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$

Izotermno sabijanje $T = \text{const}$

$$\Delta S = \int_{S_1}^{S_2} dS = \int_{S_1}^{S_2}\left(\frac{\partial S}{\partial T}\cancel{dT} + \frac{\partial S}{\partial p}dp\right)$$

$$= \int_{p_1}^{p_2} -\left(\frac{\partial V}{\partial T}\right)_p dp$$

$$= -\int_{p_1}^{p_2} \left(a+bp+cp^2\right) dp$$

$$= -\left[a\left(p_2-p_1\right) + b\,\frac{p_2^2-p_1^2}{2} + c\,\frac{p_2^3-p_1^3}{3}\right]$$

ZA ispit ili kolokvijum! Da li je moguća (razumna) varijacija na temu ovog zadatka? →

10. Pokazati da su infinitezimalne promene rada i količine toplote za termomehanički sistem nepotpuni (ne-egzaktni) diferencijali.

W, Q

Pretpostavimo da se rad (infinitezimalna promena) može zapisati kao dW

$$dW = +p\,dV = +p\left[\left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_T dp\right]$$

$V = V(p,T)$ $$= p\left(\frac{\partial V}{\partial T}\right)_p dT + p\left(\frac{\partial V}{\partial p}\right)_T dp$$

Uslov egzaktnog diferencijala za dW glasi

$$\left(\frac{\partial}{\partial p}\left(p\left(\frac{\partial V}{\partial T}\right)_p\right)\right)_T = \left(\frac{\partial}{\partial T}\left(\left(p\frac{\partial V}{\partial p}\right)_T\right)\right)_p$$

$$\left(\frac{\partial V}{\partial T}\right)_p + p\,\frac{\partial^2 V}{\partial p\,\partial T} = p\,\frac{\partial^2 V}{\partial T\,\partial p}$$

Da bi dW bilo egzaktno, trebalo bi da bude $\left(\frac{\partial V}{\partial T}\right)_p = 0$, što nije ispunjeno u opštem slučaju. Dakle, nije dW, već δW.

$$dU = dQ - \delta W = dQ - p\,dV$$

$U = U(V,T)$

$$\left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT = dQ - p\,dV$$

$$dQ = \left(\left(\frac{\partial u}{\partial v}\right)_T + p\right) dv + \left(\frac{\partial u}{\partial T}\right)_v dT$$

Uslov za egzaktnost diferencijala dQ glasi

$$\left(\frac{\partial}{\partial T}\left(\left(\frac{\partial u}{\partial v}\right)_T + p\right)\right)_v = \left(\frac{\partial}{\partial v}\left(\frac{\partial u}{\partial T}\right)_v\right)_T$$

$$\frac{\partial^2 u}{\partial T \partial v} + \left(\frac{\partial p}{\partial T}\right)_v = \frac{\partial^2 u}{\partial v \partial T}$$

Da bi bilo $\oint dQ = 0$, trebalo bi da bude ispunjeno da je

$$\left(\frac{\partial p}{\partial T}\right)_v = 0,$$ što nije ispunjeno u opštem slučaju. Dakle, nije dQ, već δQ.

Q.E.D.

11 Jedan mol idealnog gasa na početnoj temperaturi T_0 promeni zapreminu sa V_0 na $2V_0$ a) pri konstantnoj temperaturi, b) pri konstantnom pritisku. Naći rad koji je gas izvršio pri širenju i apsorbovanu količinu toplote pri tome.

---

1 mol , $pV = RT$

$\delta W = p\,dV$

a) $T = const$ $\quad T_0$ početno

$$\Delta W = \int_A^B p\,dV = RT_0 \int_{V_0}^{2V_0} \frac{dV}{V} = RT_0 \ln 2$$

Idealni gas $\quad U = \frac{3}{2} RT = U(T)$

$$du = \left(\frac{\partial u}{\partial T}\right)_V dT + \cancel{\left(\frac{\partial u}{\partial V}\right)_T}^{\,0} dV$$

$T = const \Rightarrow du = 0 \Rightarrow \Delta U = 0$

$$\Delta Q = \Delta u + \Delta W = \Delta W = RT_0 \ln 2$$

b) $\Delta W = \int_{V_0}^{2V_0} p\,dV = p(2V_0 - V_0) = pV_0 = RT_0$

$pV = RT$, $p = const \Rightarrow \frac{p}{R} dV = dT$

$du = \frac{3}{2} R\,dT = \frac{3}{2} p\,dV$

$\Delta u = \frac{3}{2} p \int_{V_1}^{V_2} dV = \frac{3}{2} pV_0 = \frac{3}{2} RT_0$ $\Bigg\}$ $\Delta Q = \Delta u + \Delta W = \frac{5}{2} RT_0$

12 Pokazati da za idealni gas važi Mejerova relacija

$$C_p = C_v + nR$$

gde je n broj molova idealnog gasa.

$$dU = \delta Q - \delta W = \delta Q - p\,dV$$

$$\delta Q = dU + p\,dV$$

Za idealni gas $dU = C_v\,dT$

$$\delta Q = C_v\,dT + p\,dV$$

$$= C_v\,dT + d(pV) - V\,dp$$

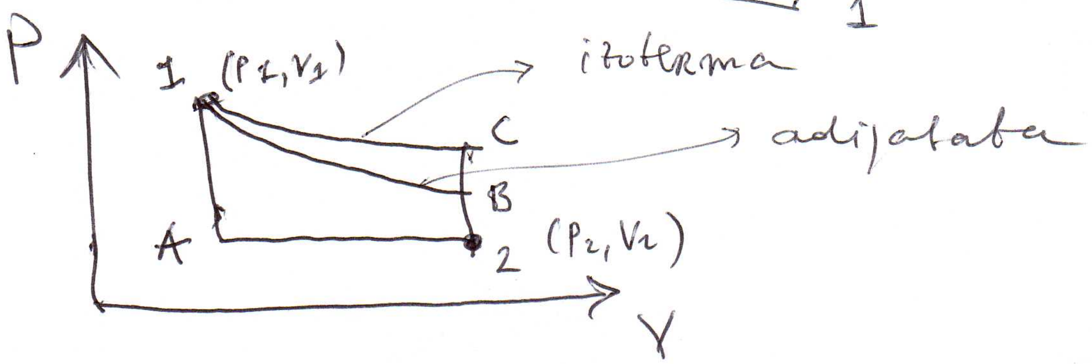
$$\left[\; pV = nRT \Rightarrow d(pV) = nR\,dT \;\right]$$

$$= (C_v + nR)\,dT - V\,dp$$

$$C_p = \left(\frac{\delta Q}{dT}\right)_p \Rightarrow C_p = C_v + nR$$

$$\boxed{C_p - C_v = nR}$$

13. Idealni gas sa termičkom i kaloričkom jednačinom stanja $pV = NkT$ i $U = \frac{3}{2} NkT$, respektivno, prevodi se reverzibilno iz stanja 1 u stanje 2 na tri različita načina. Naći u sva tri slučaja rad koji je sistem izvršio, toplotu koju je sistem primio i promenu unutrašnje energije



$\boxed{W}$

Izoterma

$$pV = NkT \Rightarrow p = \frac{NkT}{V}$$

$$\delta W = p\,dV \Rightarrow W = \int_{V_1}^{V_2} NkT \frac{dV}{V}$$

$$= NkT_1 \int_{V_1}^{V_2} \frac{dV}{V} = NkT_1 \ln \frac{V_2}{V_1}$$

$$= p_1 V_1 \ln \frac{V_2}{V_1} = p_2 V_2 \ln \frac{V_2}{V_1}$$

Adijabata $\delta Q = 0$

$$dU = \delta Q - \delta W = -\delta W$$

$$\delta W = -dU$$

Pravougla putanja

$$W = \int_1^2 p\,dV = P_2 (V_2 - V_1)$$

$\boxed{U}$

U je f-ja stanja

$$\Delta U = \int_1^2 dU = \int_1^2 \frac{3}{2} Nk\,dT = \frac{3}{2} Nk (T_2 - T_1)$$

$$= \frac{3}{2} (P_2 V_2 - P_1 V_1)$$

$\boxed{Q}$

$$Q = \Delta U + W$$

Izoterma

$$Q = \frac{3}{2} (P_2 V_2 - P_1 V_1) + P_2 V_2 \ln \frac{V_2}{V_1}$$

Adijabata

$$Q = 0$$

Pravougla putanja

$$Q = \frac{3}{2}\left(p_2 V_2 - p_1 V_1\right) + p_2 (V_2 - V_1)$$

Domaći

Za slučaj adijabatske putanje, izračunati rad koristeći j-nu adijabate!

$$\delta Q = C_v dT + p dV, \qquad \delta Q = 0$$

$$pV = nRT$$

$$0 = C_v dT + \frac{nRT}{V} dV \quad / : T$$

$$0 = C_v \frac{dT}{T} + \frac{nR}{V} dV \quad / \int, \qquad C_p - C_v = nR, \quad C_p / C_v = k$$

$$T V^{k-1} = const \Rightarrow \quad p V^{\varkappa} = const$$

$$p V^{\varkappa} = p_1 V_1^{\varkappa}$$

$$W = \int_{V_1}^{V_2} p\, dV = \int_{V_1}^{V_2} p_1 V_1^{\varkappa} \frac{dV}{V^{\varkappa}}$$

$$= \frac{p_1 V_1^{\varkappa}}{\varkappa - 1}\left(V_1^{1-\varkappa} - V_2^{1-\varkappa}\right) = \frac{1}{\varkappa - 1}\left(p_1 V_1 - p_2 V_2\right)$$

Iz Mejerove relacije

$$\frac{1}{\varkappa - 1} = \frac{C_v}{nR} \overset{\text{za idealni gas}}{=\!=} \frac{3}{2},$$

Pa je konačan izraz za rad:

$$W = \frac{3}{2}\left(p_1 V_1 - p_2 V_2\right)$$, a to je već naučeno!

14. Polazeći od jednačine (diferencijalne forme?)

$$T\,\underline{\underline{ds}} = \underline{\underline{du}} + p\,\underline{\underline{dV}} \qquad \underbrace{ds,\ du,\ dV}$$

proveriti relaciju

$$\frac{\partial(P,V)}{\partial(T,S)} = 1$$

i iskoristiti je za dobijanje Meksvelovih termodinamičkih j-na.

$$S = S(x,y) \quad \rightarrow \quad ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy$$

$$V = V(x,y) \qquad dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy$$

totalni diferencijali

$$u = u(x,y)$$

$$du = T\,ds - p\,dV$$

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = T\left(\frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy\right) - p\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy\right)$$

$$\frac{\partial u}{\partial x} = T\frac{\partial s}{\partial x} - p\frac{\partial V}{\partial x}$$

$$\frac{\partial u}{\partial y} = T\frac{\partial s}{\partial y} - p\frac{\partial V}{\partial y}$$

dU totalni diferencijal $\Rightarrow$ $\dfrac{\partial^2 u}{\partial x\partial y} = \dfrac{\partial^2 u}{\partial y\partial x}$

$$\frac{\partial^2 U}{\partial y\partial x} = \frac{\partial T}{\partial y}\frac{\partial s}{\partial x} + T\frac{\partial^2 s}{\partial y\partial x} - \frac{\partial p}{\partial y}\frac{\partial V}{\partial x} - p\frac{\partial^2 V}{\partial y\partial x}$$

$$\frac{\partial^2 U}{\partial x\partial y} = \frac{\partial T}{\partial x}\frac{\partial s}{\partial y} + T\frac{\partial^2 s}{\partial x\partial y} - \frac{\partial p}{\partial x}\frac{\partial V}{\partial y} - p\frac{\partial^2 V}{\partial x\partial y}$$

$$\frac{\partial T}{\partial y}\frac{\partial S}{\partial x}-\frac{\partial P}{\partial y}\frac{\partial V}{\partial x}=\frac{\partial T}{\partial x}\frac{\partial S}{\partial y}-\frac{\partial P}{\partial x}\frac{\partial V}{\partial y}$$

$$\frac{\partial T}{\partial y}\frac{\partial S}{\partial x}-\frac{\partial T}{\partial x}\frac{\partial S}{\partial y}\equiv\frac{\partial P}{\partial y}\frac{\partial V}{\partial x}-\frac{\partial P}{\partial x}\frac{\partial V}{\partial y}$$

$$\frac{\partial(T,S)}{\partial(y,x)}=\frac{\partial(P,V)}{\partial(y,x)}\Big/\frac{\partial(y,x)}{\partial(P,V)}$$

$$\frac{\partial(T,S)}{\partial(\cancel{y},\cancel{x})}\,\frac{\partial(y,x)}{\partial(P,V)}=\underbrace{\frac{\partial(P,V)}{\partial(y,x)}\,\frac{\partial(y,x)}{\partial(P,V)}}_{1}$$

$$\frac{\partial(T,S)}{\partial(P,V)}=1\Rightarrow\frac{\partial(P,V)}{\partial(T,S)}=1$$

Meksvelove jednačine slede iz

$$\frac{\partial(T,S)}{\partial(y,x)}=\frac{\partial(P,V)}{\partial(y,x)}$$

$\boxed{x=S \quad y=V}$ $\rightarrow$

$$\frac{\partial(T,S)}{\partial(V,S)}=\frac{\partial(P,V)}{\partial(V,S)}$$

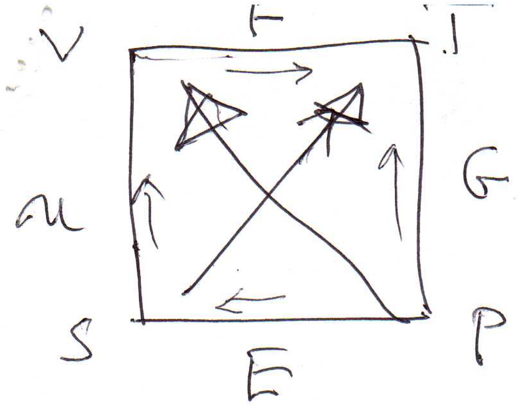
$$\boxed{\left(\frac{\partial T}{\partial V}\right)_S=-\left(\frac{\partial P}{\partial S}\right)_V}$$

$x=S \quad y=P$

$x=T \quad y=P$

$x=T \quad y=V$

Domaći: Završiti za ostale parove



$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

MNEMONIČKI

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

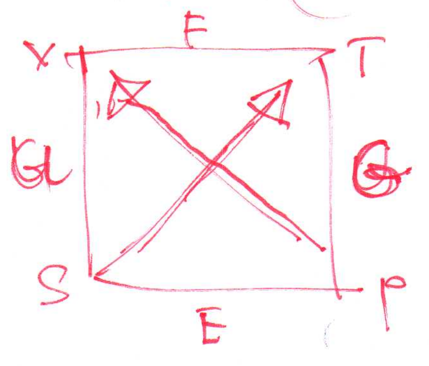
$$\left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial T}{\partial P}\right)_S$$

Primedba: Za kolokvijum

Koristeći osobine Jakobijana i Meksvelove TD j-ne pokazati da važi

$$\frac{\partial(P,V)}{\partial(T,S)} = 1$$

$$\frac{\partial(P,V)}{\partial(S,V)} \frac{\partial(S,V)}{\partial(T,S)} = -\left(\frac{\partial P}{\partial S}\right)_V \left(\frac{\partial V}{\partial T}\right)_S$$



$$\Rightarrow \left(\frac{\partial P}{\partial S}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_S$$

$$\frac{\partial(P,V)}{\partial(T,S)} = 1$$

4. Koeficijent izobarnog termičkog širenja α_p, za vodu, je negativan pri temperaturama $0^\circ C \leq t \leq 4^\circ C$. Pokazati da će u tom intervalu temperatura reverzibilno adijabatsko sabijanje vode imati za posledicu njeno hlađenje a ne zagrevanje.

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

adijabatski proces $\quad \delta Q = 0$

$$\delta Q = TdS$$

$$TdS = dU + pdV \quad ; \quad U = U(V,T)$$

$$= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + pdV$$

$$\left(\frac{\partial U}{\partial V}\right)_T = -p + T\left(\frac{\partial p}{\partial T}\right)_V \; ; \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\partial(p,V)}{\partial(T,V)} = \frac{\dfrac{\partial(p,V)}{\partial(p,T)}}{\dfrac{\partial(T,V)}{\partial(p,T)}} = \frac{V\alpha_p}{\cancel{V}\, K_T}$$

$$\left[K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \sim \text{izotermska kompresibilnost} \right]$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\alpha_p}{\kappa_T}$$

$$-C_V\, dT = T\frac{\alpha_p}{\kappa_T}\, dV$$

$$dT = \underbrace{-\frac{T\alpha_p}{C_V \kappa_T}}_{>0}\, dV \Rightarrow dV < 0$$

$$\Downarrow$$

$$dT < 0$$

$$\left.\begin{matrix} C_p \\ C_V \\ \kappa_T \\ \kappa_S \end{matrix}\right\} > 0$$

$$\alpha_p \gtrless 0$$

16

Pokazati da ako

1. Sistem NE vrši rad ($V=const$) i nalazi se na konstantnoj temperaturi onda stanju TD. ravnoteže odgovara minimum slobodne energije

2. Ako je $p=const$, $T=const$ onda stanju TD ravnoteže odgovara minimum Gibsovog potencijala

---

1.

I zakon TD

$$\Delta U + \Delta W = \Delta Q$$

$$\int_A^B \frac{\delta Q}{T} \leq S(B) - S(A) \Rightarrow \Delta Q \leq T\Delta S \Rightarrow$$

$$\Delta U + \cancel{\Delta W}_0 - T\Delta S \leq 0 \Rightarrow \Delta(U - TS) \leq 0$$

$$\Delta F \leq 0$$

2. $\Delta U + \Delta W = \Delta U + p\Delta V = \Delta Q \leq T\Delta S \Rightarrow$

$$\Delta U - T\Delta S + pdV \leq 0 \qquad \Delta(U - TS + pV) \leq 0$$

$$\Delta G \leq 0$$

Domaći:

$S=const$, $p=const$ u TDR $\Rightarrow \Delta E \leq 0$, $S=const$, $V=const \Rightarrow \Delta U \leq 0$

11. Ispitati kako se C_p menja sa pritiskom i C_v sa zapreminom pri izotermnim reverzibilnim procesima, za gasove čije su termičke j-ne stanja date kao:

a. $pV = RT$

b. $\left(p + \frac{a}{V^n}\right)(V-b) = RT$

c. $p(V-b) = RT\, e^{-\frac{a}{RTV}}$

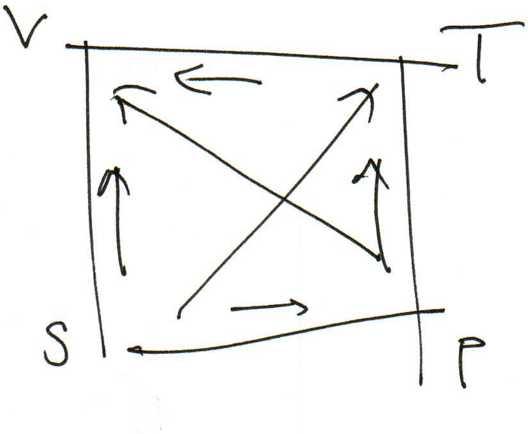
uraditi!

gde su a, b, n i R konstante

Računamo $\left(\frac{\partial C_v}{\partial V}\right)_T$ i $\left(\frac{\partial C_p}{\partial p}\right)_T$

$$\left(\frac{\partial C_p}{\partial p}\right)_T = \frac{\partial}{\partial p}\left(T\left(\frac{\partial S}{\partial T}\right)_p\right)_T = T\frac{\partial^2 S}{\partial p\,\partial T} = T\frac{\partial^2 S}{\partial T\,\partial p}$$

$$= T\left[\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial p}\right)_T\right]_p$$



$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

$$= -T\left[\frac{\partial^2 V}{\partial T^2}\right]_p$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \frac{\partial}{\partial V}\left(T\left(\frac{\partial S}{\partial T}\right)_V\right)_T = T\frac{\partial^2 S}{\partial V\,\partial T} = T\frac{\partial^2 S}{\partial T\,\partial V}$$

$$= T\left[\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right]_V = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

a.

$$\left(\frac{\partial C_p}{\partial p}\right)_T = -T\left[\frac{\partial^2 V}{\partial T^2}\right]_p$$

$$pV = RT \Rightarrow V = \frac{RT}{p}$$

$$\frac{\partial V}{\partial T} = \frac{R}{p} \qquad \frac{\partial^2 V}{\partial T^2} = 0$$

$$\left(\frac{\partial C_p}{\partial p}\right)_T = 0$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left[\frac{\partial^2 p}{\partial T^2}\right]_V$$

$$pV = RT \Rightarrow p = \frac{RT}{V} \Rightarrow \frac{\partial p}{\partial T} = \frac{R}{V}, \quad \frac{\partial^2 p}{\partial T^2} = 0$$

b.
c. } Domaći

~~- Dobiti izraze za $\left(\frac{\partial C_V}{\partial V}\right)_T$ i $\left(\frac{\partial C_p}{\partial p}\right)_T$ iz funkcionalnih veza $S = S(T,V)$ i $S = S(T,p)$ koristeći Maxwellove jne i uslov da je dS totalni diferencijal~~

NE!

20

Termička i kalorička j-na stanja za klasični realni gas (1 mol gasa) su date izrazima:

$$\left(p+\frac{a}{V^2}\right)(V-b) = RT$$

i

$$U = \frac{3}{2}RT - \frac{a}{V}$$

respektivno. Ako je poznato da važi veza

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$$

naći C_p i C_v za ovaj gas. Onda naći $C_p - C_v$ za idealan gas.

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_V = \frac{3}{2}R$$

$$\left(\frac{\partial p}{\partial T}\right)_V = ?$$

$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = ?$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{\left(\frac{\partial T}{\partial V}\right)_p}$$

$$T = \frac{1}{R}\left(p + \frac{a}{V^2}\right)(V-b)$$

$$\left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{R}\left[-\frac{2a}{V^3}(V-b) + \left(p + \frac{a}{V^2}\right)\right]$$

$$\left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{R}\left[\frac{2ab}{V^3} - \frac{2a}{V^2} + p + \frac{a}{V^2}\right]$$

$$\left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{R}\,\frac{pV^3 + aV - 2aV + 2ab}{V^3}$$

$$\left(\frac{\partial T}{\partial V}\right)_p = \frac{1}{R}\,\frac{pV^3 - aV + 2ab}{V^3}$$

$$\frac{1}{\left(\frac{\partial T}{\partial V}\right)_p} = \frac{RV^3}{pV^3 - aV + 2ab} = \left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p = C_V + T\,\frac{R}{V-b}\,\frac{RV^3}{pV^3 - aV + 2ab}$$

$$C_p = \frac{3}{2}R + \frac{TR}{V-b}\,\frac{RV^3}{pV^3 - aV + 2ab}$$

$C_p - C_v$ za idealni gas

$a \to 0$

$b \to 0$ J-na stanja za stvarni realni gas $\Longrightarrow$ J-ne stanja za idealni gas

$$C_p - C_v = T \frac{R}{V-b} \frac{RV^3}{pV^3 - aV + 2ab}$$

$$\lim_{\substack{a \to 0 \\ b \to 0}} (C_p - C_v) = T \frac{R}{V} \frac{R}{p} \overset{pV=RT}{=\!=} T \frac{R^2}{RT} = R$$

$$(C_p - C_v)_{\text{Idealni gas}} = R$$ Majerova relacija

Mayer-ova relacija

Ili

$pV = RT$

$U = \frac{3}{2} RT$

$$C_v = \left(\frac{\partial u}{\partial T}\right)_V = \frac{3}{2} R$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p}$$

Domaći

$$C_p - C_v = T \frac{R^2}{pV} = \frac{TR^2}{TR} = R$$

$$\boxed{C_p - C_v = R}$$ 1 mol gasa

7] Naći razliku molarnih specifičnih toplota $C_p - C_V$ za gas koji se pokorava jednoj od navedenih j-na stanja

a) $pV = RT$ (idealni gas)

b) $\left(p + \frac{a}{V^n}\right)(V - b) = RT$ (Van der Waals-ova)

c) $p(V-b) = RT e^{-\frac{a}{RTV}}$. (Dieterici)

a, b, n i R su konstante. Koristiti izraz

$$C_p - C_V = T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p .$$

---

a) i b) Domaći

a)

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V} \quad , \quad \left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p}$$

$$C_p - C_V = \frac{TR^2}{pV} = R$$

b)

$$p = \frac{RT}{V-b} - \frac{a}{V^n} \Rightarrow \left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = ? \qquad \left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{\left(\frac{\partial T}{\partial V}\right)_p}$$

$$T = \frac{1}{R}\left(pV - pb + \frac{a}{V^{n-1}} - \frac{ab}{V^n}\right)$$

način

$$c_p - c_v = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial v}{\partial T}\right)_p$$

$$p(v-b) = RT\, e^{-\frac{a}{RTV}} \longrightarrow p = \frac{RT}{V-b}\, e^{-\frac{a}{RTV}}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b}\left[e^{-\frac{a}{RTV}} + T\left(-\frac{a}{RTV}\right)' e^{-\frac{a}{RTV}} \right]$$

$$= \frac{R}{V-b}\, e^{-\frac{a}{RTV}} \left[1 + T\,\frac{a}{RT^2V} \right]$$

$$= \frac{R}{V-b}\, e^{-\frac{a}{RTV}} \left[1 + \frac{a}{RTV} \right]$$

$$\left(\frac{\partial v}{\partial T}\right)_p = ?$$

Euler-ovo ciklično pravilo

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$$

$$\Downarrow$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial T}{\partial p}\right)_V} = -\frac{\left(\frac{\partial p}{\partial T}\right)_V}{\left(\frac{\partial p}{\partial V}\right)_T}$$

Dakle, umesto $\left(\frac{\partial V}{\partial T}\right)_p$, tražimo $\left(\frac{\partial p}{\partial v}\right)_T$

$$\left(\frac{\partial p}{\partial V}\right)_T = RT\left[-\frac{1}{(V-b)^2}\,e^{-\frac{a}{RTV}} + \frac{1}{V-b}\left(-\frac{a}{RTV}\right)' e^{-\frac{a}{RTV}}\right.$$

$$= RT\,e^{-\frac{a}{RTV}}\left[-\frac{1}{(V-b)^2} + \frac{1}{V-b}\,\frac{a}{RTV^2}\right]$$

$$= RT\,e^{-\frac{a}{RTV}}\,\frac{-RTV^2 + a(V-b)}{(V-b)^2\,RT\,V^2}$$

$$= e^{-\frac{a}{RTV}}\,\frac{a(V-b) - RTV^2}{V^2(V-b)^2}$$

$$\frac{1}{\left(\frac{\partial p}{\partial V}\right)_T} = e^{\frac{a}{RTV}}\,\frac{V^2(V-b)^2}{a(V-b) - RTV^2}$$

$$C_p - C_V = -T\,\frac{\left(\frac{\partial p}{\partial T}\right)_V^2}{\left(\frac{\partial p}{\partial V}\right)_T} = -T\,\frac{R^2}{\cancel{(V-b)^2}}\,e^{-\frac{2a}{RTV}}\left[1+\frac{a}{RTV}\right]^2 \cdot e^{\frac{a}{RTV}}\,\frac{V^2\cancel{(V-b)^2}}{a(V-b) - RTV^2}$$

$$C_p - C_V = TR^2\,e^{-\frac{a}{RTV}}\left[1+\frac{a}{RTV}\right]^2\,\frac{V^2}{RTV^2 - a(V-b)}$$

Iz Diteričijeve j-ne

$$V-b = \frac{RT}{p}\,e^{-\frac{a}{RTV}}$$

Ovde može lomes, ne mora iz dalje.

20. Pokazati da se razlika specifičnih toplota $C_p - C_V$ može dovesti u vezu sa koeficijentima α_p i κ_T.

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$$

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

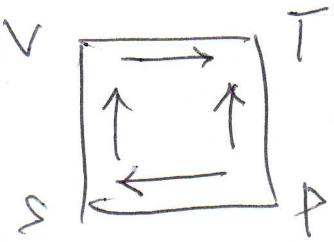
$$S = S(T, V) \qquad V = V(p, T)$$

$$S = S(T, V(p, T))$$

$$\left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

$$T\left[\left(\frac{\partial S}{\partial T}\right)_p - \left(\frac{\partial S}{\partial T}\right)_V\right] = T\left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

$$\boxed{C_p - C_V = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p} \qquad (*)$$



$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

Euler-ova ciklična relacija

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1 \qquad (**)$$

Iz (**)

$$\left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

Smenom u (*) dobija se

$$C_p - C_V = -T \frac{\left(\frac{\partial V}{\partial T}\right)_p^2}{\left(\frac{\partial V}{\partial p}\right)_T} = VT \frac{\alpha_p^2}{\kappa_T}$$

21. Pokazati da iz III zakona termodinamike sledi da toplotni kapacitet teži nuli kada temperatura teži nuli.

$$C_x = T\left(\frac{\partial S}{\partial T}\right)_x$$

III zakon TD: $\lim\limits_{T\to 0} S = 0$

$$\lim_{T\to 0} S = \lim_{T\to 0}\frac{TS}{T} = \lim_{T\to 0}\frac{\left(\frac{\partial (TS)}{\partial T}\right)_x}{\left(\frac{\partial T}{\partial T}\right)_x}$$

$$= \lim_{T\to 0}\left(S + T\left(\frac{\partial S}{\partial T}\right)_x\right) = \lim_{T\to 0} S + \lim_{T\to 0} C_x \Rightarrow$$

$$\lim_{T\to 0} C_x = 0 \quad , \forall x$$

Primedba: Primetiti primenu Lopitalovog pravila

L'Hospital

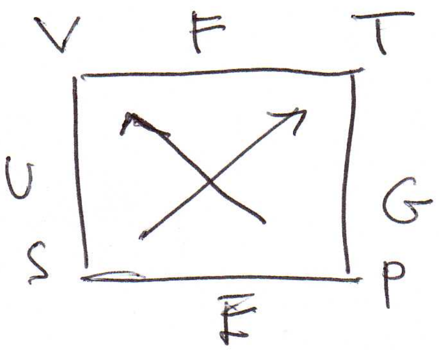
22. Pokazati važenje relacije

$$\left(\frac{\partial U}{\partial N}\right)_{T,V} - \mu = -T\left(\frac{\partial \mu}{\partial T}\right)_{V,N}.$$

Sistem se sastoji od N čestica, a V, T, U i μ su zapremina sistema, temperatura, unutrašnje energija i hemijski potencijal, respektivno.

(Kubo, 163. str.)

$$dF = -p\,dV - S\,dT + \mu\,dN \qquad (*)$$



$$(*) \Rightarrow \quad \mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} \qquad S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}$$

$$\boxed{F = U - TS} \quad \Big/ \frac{\partial}{\partial N}$$

$$\left(\frac{\partial F}{\partial N}\right)_{T,V} = \left(\frac{\partial U}{\partial N}\right)_{T,V} - T\left(\frac{\partial S}{\partial N}\right)_{T,V}$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{T,V} + T\left(\frac{\partial^2 F}{\partial N \partial T}\right)_V$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{T,V} + T\left[\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial N}\right)_T\right]_V$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{T,V} + T\left(\frac{\partial \mu}{\partial T}\right)_{V,N} \quad , \text{ odnosno}$$

$$\left(\frac{\partial U}{\partial N}\right)_{T,V} = \mu - T\left(\frac{\partial \mu}{\partial T}\right)_{V,N}$$

Neholonomna veza između U i μ koja podseća na vezu kalorične i termičke j-ne.

23. Kod izotermnog zračenja gustina unutrašnje energije u je monotono rastuća f-ja temperature a pritisak je $p=\frac{1}{3}u$. Kakav oblik funkcionalne zavisnosti $u(T)$ predviđa na osnovi ovih podataka fenomenološka termodinamika? Da li je rezultat saglasan sa zakonima zračenja? Za izotermno zračenje naći izraz za entropiju ravnotežnog zračenja, njegove TD potencijale i specifične toplote ξ_p i ξ_V obračunate po jedinici zapremine šupljine ispunjene ravnotežnim zračenjem.

$$\frac{du}{dT}>0,\quad u=u(T),\quad p=\frac{1}{3}u$$

$$TdS = dU + pdV$$

$$U = V\cdot u$$

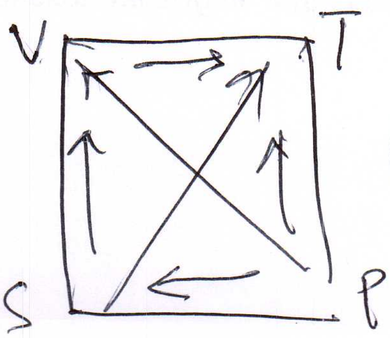
$$TdS = udV + Vdu + pdV$$

$$dS = \frac{1}{T}(u+p)dV + \frac{V}{T}du$$

$$dS = \frac{4u}{3T}dV + \frac{Vdu}{T}$$

$$\Downarrow$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T}$$



$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{4u}{3T} \Rightarrow \frac{1}{3}\frac{du}{dT} = \frac{4}{3}\frac{u}{T}$$

$$\frac{du}{u} = 4\frac{dT}{T} \longrightarrow \boxed{u = aT^4}$$

Stefan-Bolcmanov zakon

(kalorička j-na stanja)

$$ds = \frac{4}{3T}\, aT^4 dV + 4aT^3 V dT$$

$$ds = 4a\left(\frac{T^3}{3}dV + T^2 V dT\right)$$

$$d\left(\frac{T^3 V}{3}\right) = T^2 V dT + \frac{1}{3}T^3 dV$$

$$ds = 4a\, d\left(\frac{T^3 V}{3}\right)$$

$$ds = d\left(\frac{4aT^3V}{3}\right) \Rightarrow S = \frac{4aT^3V}{3} + S_0$$

III закон TД : $\lim_{T\to 0^+} S = 0 \Rightarrow S_0 = 0$

$$\boxed{S = \frac{4aT^3V}{3}}$$

$$\mathcal{U} = uV = aT^4V = aV\left(\frac{3S}{4aV}\right)^{\frac{4}{3}} = U(S,V)$$

$$F = U - TS = aT^4V - T\frac{4}{3}aT^3V = -\frac{1}{3}aT^4V = F(T,V)$$

$$G = U - TS + pV = F + pV = -\frac{1}{3}aT^4V + \frac{1}{3}aT^4V$$

$$= 0$$

$$\Downarrow$$

$$G = \mu N \Rightarrow \boxed{\mu = 0}$$

hemijski potencijal za ravnotežno termalno zračenje

Kako je $G = 0$ i $G = E - TS \Rightarrow$

$$E = TS = \left(\frac{3p}{a}\right)^{\frac{1}{4}} S$$

$$\xi_V = \frac{C_V}{V} = \frac{1}{V}\left(\frac{\partial \mathcal{U}}{\partial T}\right)_V = \frac{1}{V}4aT^3V = 4aT^3$$

$$\xi_p = \frac{C_p}{V} = \frac{1}{V}T\left(\frac{\partial S}{\partial T}\right)_p \to \infty$$ jer

$$p = \frac{1}{3}aT^4 \Leftrightarrow p = \text{const},\ T = \text{const}$$

Za kolokvijum
Koristeći vezu

$$\left(\frac{\partial \mathcal{U}}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

potvrditi važenje Stefan–Bolcmanovog zakona

24

Za magnetne materijale i reverzibilne procese I i II zakon TD se izražavaju kroz diferencijalnu formu:

$$Tds = du - HdM$$

ukoliko su promene zapremine zanemarljive. Ovde je H jačina magnetnog polja a M je magnetizacija (magnetni moment sistema). Uvodeći specifične toplotne kapacitete pri konstantnoj magnetizaciji, c_M, i pri konstantnom magnetnom polju, c_H, dokazati da između adijabatske susceptibilnosti sistema, $\chi_S = \left(\frac{\partial M}{\partial H}\right)_S$ i izotermne susceptibilnosti, $\chi_T = \left(\frac{\partial M}{\partial H}\right)_T$, postoji veza:

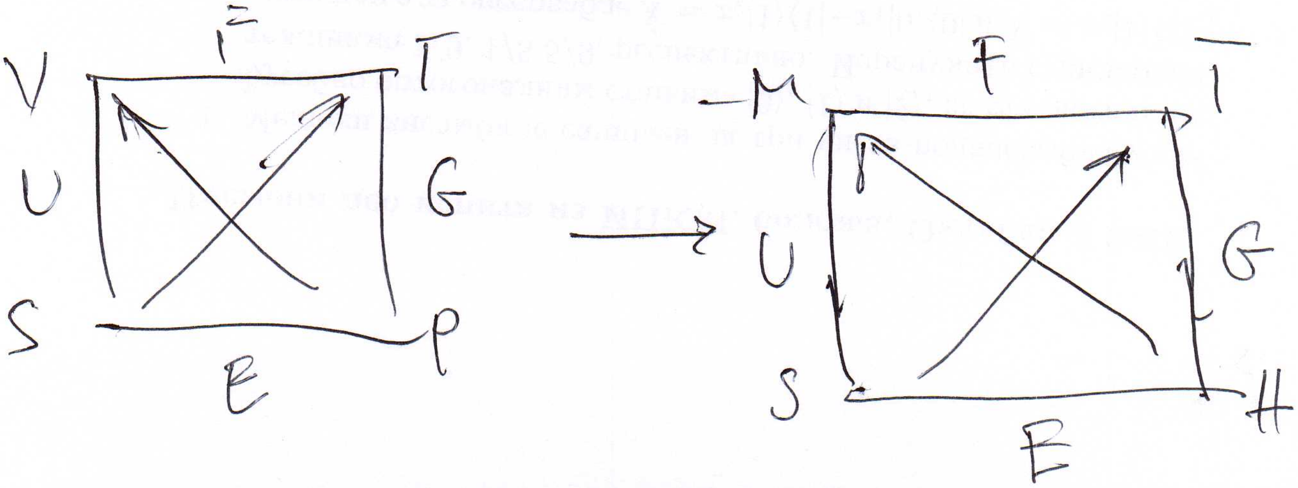
$$\chi_S = \frac{c_M}{c_H}\chi_T$$

$$Tds = du - HdM$$

$$Tds = du + pdV$$

$$V \longleftrightarrow -M$$

$$P \longleftrightarrow H$$



$$C_H = T\left(\frac{\partial S}{\partial T}\right)_H, \qquad C_M = T\left(\frac{\partial S}{\partial T}\right)_M$$

$$\frac{C_M}{C_H} = \frac{\dfrac{\partial(S,M)}{\partial(T,M)}}{\dfrac{\partial(S,H)}{\partial(T,H)}} = \frac{\partial(T,H)}{\partial(S,H)}\,\frac{\partial(S,M)}{\partial(T,M)}$$

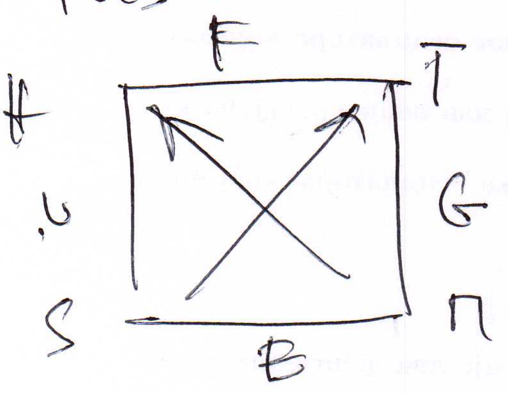
$$= \frac{\partial(T,H)}{\partial(T,M)}\,\frac{\partial(S,M)}{\partial(S,H)}$$

$$= \left(\frac{\partial H}{\partial M}\right)_T \left(\frac{\partial M}{\partial H}\right)_S = \frac{\left(\frac{\partial M}{\partial H}\right)_S}{\left(\frac{\partial M}{\partial H}\right)_T} = \frac{\chi_S}{\chi_T}$$

Vežba: $\dfrac{\partial(S,T)}{\partial(-M,H)} = 1$

Alternativa

$$T\,dS = dU + M\,dH$$



$P \longleftrightarrow M$

$V \longleftrightarrow H$

Pokazati $\dfrac{\partial(S,T)}{\partial(H,M)} = 1$

Zapisati dif. forme za preostale TD potencijale i odgovarajući skup Maksvelovih relacija

Pogledati zadatak (2.22) u Z.Z. kada je promena zapremine od interesa

25 Znajući da važi

$$C_p - C_V = T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$$

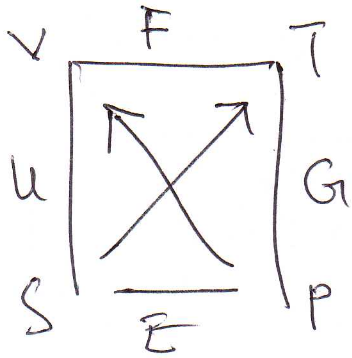
pokazati da za termomehaničke sisteme važe sledeće relacije:

a) $$C_p - C_V = \frac{T \left(\frac{\partial^2 F}{\partial T \partial V}\right)^2}{\left(\frac{\partial^2 F}{\partial V^2}\right)_T}$$

b) $$\left(\frac{\partial S}{\partial T}\right)_V - \left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial S}{\partial p}\right)_T$$

d)

$$C_p - C_V = T\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_V$$



$$dF = -p\,dV - S\,dT$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$f(p, V, T) = 0 \quad \Rightarrow$$

J-na Stanja

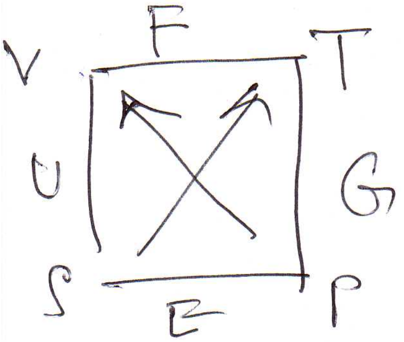
$$\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T = -1$$ (Euler)

$$\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T} = -\frac{\left(\frac{\partial p}{\partial T}\right)_V}{\left(\frac{\partial p}{\partial V}\right)_T}$$

$$C_p - C_V = -T\,\frac{\left(\frac{\partial p}{\partial T}\right)_V^2}{\left(\frac{\partial p}{\partial V}\right)_T} = \cancel{-}T\,\frac{\left(\frac{\partial^2 F}{\partial T \partial V}\right)^2}{-\left(\frac{\partial^2 F}{\partial V^2}\right)_T}$$

$$C_p - C_V = T\,\frac{\left(\frac{\partial^2 F}{\partial T \partial V}\right)^2}{\left(\frac{\partial^2 F}{\partial V^2}\right)_T}$$

b) $C_p - C_V = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$



$$\longrightarrow \left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$T\left[\left(\frac{\partial S}{\partial T}\right)_p - \left(\frac{\partial S}{\partial T}\right)_V\right] = -T\left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial S}{\partial p}\right)_T$$

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_V - \left(\frac{\partial S}{\partial T}\right)_p = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial S}{\partial p}\right)_T}$$

26. Извести израз за адијабатску компресибилност идеалног гаса који се квазистационарно и адијабатски сабија. Потом извести израз за брзину звука користећи дефиницију

$$v = \sqrt{\frac{dp}{d\rho}}$$

где је p притисак а ρ густина гаса.

Једначина адијабатског процеса

$$pV^{\varkappa} = const \qquad \varkappa = \frac{C_p}{C_V}$$

Подсетник

$$\delta Q = C_V dT + p dV, \qquad \text{адијабатика} \quad \delta Q = 0$$

$$C_V dT = -p dV, \qquad p = \frac{nRT}{V}$$

$$C_V dT = -\frac{nRT}{V} dV$$

$$\frac{dT}{T} = -\frac{nR}{C_V}\frac{dV}{V}, \qquad \text{Mayer} \quad C_p - C_V = nR$$

$$\frac{dT}{T} = \frac{C_V - C_p}{C_V}\frac{dV}{V}, \qquad \frac{C_p}{C_V} = \varkappa$$

$$\frac{dT}{T} = (1-\varkappa)\frac{dV}{V}$$

$$\vdots$$

$$TV^{\varkappa-1} = const \Rightarrow pV^{\varkappa} = const$$

$$pV^{\varkappa} = const$$

$$\ln pV^{\varkappa} = const$$

$$\ln p + \ln V^{\varkappa} = const \Rightarrow \ln p + \varkappa \ln V = const \; / d$$

$$\frac{dp}{p} + \varkappa \frac{dV}{V} = 0 \Rightarrow \frac{1}{V}\frac{dV}{dp} = -\frac{1}{\varkappa p}$$

$$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

$$\Rightarrow \boxed{K_S = \frac{1}{\varkappa p}}$$

БРЗИНА ЗВУКА. Посматрајмо извод $\left(\frac{\partial p}{\partial \rho}\right)_S$ који има димензију $\left(\frac{m}{s}\right)^2$

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \left(\frac{\partial p}{\partial V}\right)_S \left(\frac{\partial V}{\partial \rho}\right)_S$$

$$\left[\rho = \frac{m}{V} \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V} \Rightarrow \frac{dV}{d\rho} = -\frac{V}{\rho} \Rightarrow \left(\frac{\partial V}{\partial \rho}\right)_S = -\frac{V}{\rho} \right]$$

$$\left(\frac{\partial p}{\partial \rho}\right)_S = -\frac{V}{\rho}\left(\frac{\partial p}{\partial V}\right)_S$$

$$= -\frac{V}{\rho}\frac{1}{\left(\frac{\partial V}{\partial p}\right)_S} = -\frac{1}{\rho}\frac{1}{\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S} = \frac{1}{\rho K_S} = \frac{\varkappa p}{\rho}$$

$$\boxed{v = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_S} = \sqrt{\frac{\varkappa p}{\rho}}}$$

БРЗИНА ЗВУКА У ~~КРОЗ~~ ИДЕАЛНОМ ГАСУ

МОЛЕКУЛАРНА ФИЗИКА

ФЧС